

Test - 4

①

- 1) Find the exact value of $\tan(75^\circ)$ by using the addition or subtraction formulas

Ans

$$C = \tan(75^\circ)$$

$$C = \tan(45^\circ + 30^\circ)$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$A = 45^\circ \text{ and } B = 30^\circ$$

$$\tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ}$$

$$\tan 45^\circ = 1 \text{ and } \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\tan 75^\circ = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\boxed{\tan 75^\circ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}}$$

$$\tan 75^\circ = \frac{(\sqrt{3} + 1)^2}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{(\sqrt{3})^2 + 2 \times \sqrt{3} \times 1 + 1^2}{(\sqrt{3})^2 - 1}$$

$$= \frac{3 + 1 + 2\sqrt{3}}{3 - 1}$$

$$\underline{\underline{\tan 75^\circ = 2 + \sqrt{3}}}$$

2. Find the exact value of the expression

$$\sin \left[\cos^{-1} \left(\frac{\sqrt{6}}{3} \right) + \tan^{-1} \left(-\frac{3}{4} \right) \right]$$

Ans

Let $\cos^{-1} \left(\frac{\sqrt{6}}{3} \right) = \alpha$ then $\cos \alpha = \frac{\sqrt{6}}{3}$

and $\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{6}{9}} = \sqrt{\frac{3}{9}} = \frac{\sqrt{3}}{3}$

Let $\tan^{-1} \left(-\frac{3}{4} \right) = \beta$, then we have $\tan \beta = \left(-\frac{3}{4} \right)$

and,

$$\cos \beta = \frac{1}{\sec \beta} = \frac{1}{\sqrt{1 + \tan^2 \beta}} = \frac{1}{\sqrt{1 + \frac{9}{16}}} = \frac{4}{5}$$

and,

$$\sin \beta = \tan \beta \times \cos \beta = \left(-\frac{3}{4} \right) \times \frac{4}{5} = \left(-\frac{3}{5} \right)$$

Hence, $\sin \left[\cos^{-1} \left(\frac{\sqrt{6}}{3} \right) + \tan^{-1} \left(-\frac{3}{4} \right) \right]$

$$= \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$= \frac{\sqrt{3}}{3} \times \frac{4}{5} + \frac{\sqrt{6}}{3} \times \left(-\frac{3}{5} \right)$$

$$= \frac{4\sqrt{3}}{15} - \frac{3\sqrt{6}}{15}$$

$$= \frac{4\sqrt{3} - 3\sqrt{6}}{15}$$

3. Road cone :

Ans

Given, $\angle A = 32^\circ$, Find AP? Assume AP = x

$$\angle B = 112^\circ \quad (\because 180^\circ - 68^\circ)$$

From angle Property of triangle, $\angle P = 180^\circ - \angle A + \angle B$

Also,

$$\angle P = 36^\circ$$

$$\frac{2}{\sin 36^\circ} = \frac{x}{\sin 112^\circ}$$

$$\Rightarrow x = 3.15 \text{ ft}$$

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the steel cord is 3.15 feet

4) use half-angle formulas to find the exact value of $\sin(157.5^\circ)$

Ans

As 157.5° is an angle in the 2nd quadrant,

Sine is positive

$$\text{Also, } \sin 157.5^\circ = \sin(180^\circ - 157.5^\circ) = \sin 22.5^\circ$$

$$\cos 2t = (1 - 2 \sin^2 t)$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = (1 - 2 \sin^2 22.5^\circ)$$

Solving for positive root,

$$\sin 22.5^\circ = \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}}$$

$$\Rightarrow \sin 22.5^\circ = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}}$$

$$5. \cos(2\beta) - \sin(\beta) = 0$$

$$\cos(2\beta) = 1 - 2\sin^2\beta$$

$$-2\sin^2\beta - \sin\beta + 1 = 0 \quad \text{in the form } ax^2 + bx + c = 0$$

Factor out -1 out of $-\sin\beta$

$$-2\sin^2\beta + (1-2)\sin\beta + 1 = 0$$

$$-2\sin^2\beta + \sin\beta - 2\sin\beta + 1 = 0$$

$$\sin\beta(-2\sin\beta + 1) + 1(-2\sin\beta + 1) = 0$$

$$(\sin\beta + 1)(-2\sin\beta + 1) = 0$$

$$\sin\beta = -1 \quad ; \quad -2\sin\beta = -1$$

$$\sin\beta = -\frac{1}{2} \Rightarrow \frac{1}{2}$$

$$\beta = \frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n, \frac{7\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n$$

for the interval $(0, 2\pi]$

$$\boxed{\beta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

6. Find the exact value of the expression.

④

$$\cos [2 \sin^{-1}(-3/4)]$$

Ans

$$\text{Let } a = \sin^{-1}(-3/4)$$

$$\text{Then } \sin a = (-3/4)$$

$$\begin{aligned}\cos 2a &= 1 - 2 \sin^2 a \\ &= 1 - 2 \left[\frac{9}{16} \right] \\ &= \underline{\underline{-\frac{1}{8}}}\end{aligned}$$

7. Find the exact value of $\tan(\frac{\theta}{2})$ for the given conditions

$$\cos \theta = \frac{\sqrt{5}}{3}; \quad 0^\circ < \theta < 90^\circ$$

Ans

We will use the identity:

$$\tan^2\left(\frac{\theta}{2}\right) = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$\tan^2\left(\frac{\theta}{2}\right) = \frac{1 - \sqrt{5}/3}{1 + \sqrt{5}/3} = \frac{\cancel{3} - 3\sqrt{5}}{\cancel{3} + 3\sqrt{5}} = \frac{3 - 3\sqrt{5}}{3 + 3\sqrt{5}} = \frac{1 - \sqrt{5}}{1 + \sqrt{5}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm \sqrt{-\frac{3 - \sqrt{5}}{2}} \quad \text{or} \quad \pm \sqrt{\frac{1 - \sqrt{5}}{1 + \sqrt{5}}}$$

But WKT, $0^\circ < \theta < 90^\circ$

$$\therefore 0 < \frac{\theta}{2} < \frac{90^\circ}{2} \quad \text{or} \quad 0 < \frac{\theta}{2} < 45^\circ$$

This means $\frac{\theta}{2}$ is in 1st quadrant, where $\tan$ is +ve

$$\therefore \tan\left(\frac{\theta}{2}\right) = \underline{\underline{\sqrt{-\frac{3 - \sqrt{5}}{2}}}} \quad \text{or} \quad \sqrt{\frac{1 - \sqrt{5}}{1 + \sqrt{5}}}$$

- 2) Solve $\triangle ABC$ by using the law of Sines if given that ⑤

Ans

$$c = 8, \alpha = 40^\circ, \gamma = 70^\circ$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$40^\circ + \beta + 70^\circ = 180^\circ$$

$$\beta = 70^\circ$$

Find a :

$$\frac{a}{\sin 40^\circ} = \frac{8}{\sin 70^\circ}$$

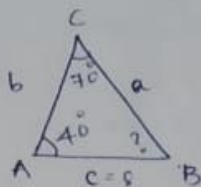
$$a = 5.47$$

Find b :

$$\frac{8}{\sin 70^\circ} = \frac{b}{\sin 70^\circ}$$

$$b = 8$$

$$a = 5.5, b = 8, \beta = 70^\circ$$



- 9) Length of Tunnel

Ans

using cosine rule,

$$c^2 = a^2 + b^2 - 2ab \cos C$$

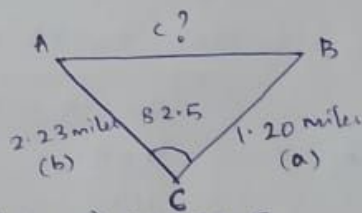
$$= (1.20)^2 + (2.23)^2 - 2(1.20)(2.23) \cos 82.5$$

$$= 1.44 + 4.97 - 0.695$$

$$c^2 = 5.712$$

$$c = 2.38 \text{ miles}$$

$$\therefore \text{Length of Tunnel} = \underline{\underline{2.38 \text{ miles}}}$$



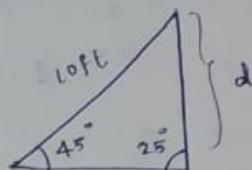
10. Installing Solar Panel
 Soln: using sine rule.

$$\frac{10}{\sin 25^\circ} = \frac{d}{\sin 45^\circ}$$

$$d = \frac{10}{\sin 25^\circ} \times \sin 45^\circ$$

$$d = 16.73 \text{ ft}$$

The length of the brace is 16.73 ft



13. Solve $\triangle ABC$ by using the law of cosine if given that
 $a=20$, $b=25$, $c=22$

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$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(25)^2 + (22)^2 - (20)^2}{2 \times 25 \times 22}$$

$$\cos A = \frac{709}{1100}$$

$$A = \cos^{-1} \left[\frac{709}{1100} \right]$$

$$A = 49.86 \approx 50$$

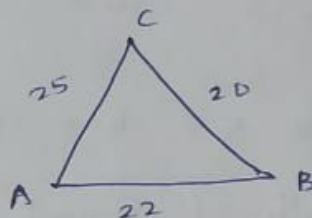
$$\cos B = \frac{c^2 + a^2 - b^2}{2ac} = \frac{(22)^2 + (20)^2 - (25)^2}{2 \times 20 \times 22} = \frac{259}{880}$$

$$B = \cos^{-1} \left[\frac{259}{880} \right] = 72.88 \approx 73$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{(20)^2 + (25)^2 - (22)^2}{2 \times 20 \times 25} = \frac{541}{1000}$$

$$C = \cos^{-1} \left[\frac{541}{1000} \right] = 57.24 \approx 57$$

$$\angle A = 50^\circ, \angle B = 73^\circ, \angle C = 57^\circ$$



$$11. \sin(3t) \cos(2t) = -\frac{\sqrt{3}}{2} + \sin(2t) \cos(3t)$$

$$\text{Let } \sin u \cos v = \frac{1}{2} (\sin(u+v) - \sin(u-v))$$

$$\sin(3t) \cos(2t) = \frac{1}{2} (\sin(5t) - \sin t)$$

$$\frac{1}{2} (\sin(5t) + \sin t) = -\frac{\sqrt{3}}{2} + \frac{1}{2} (\sin(5t) - \sin t)$$

cancel like terms: $\frac{1}{2} \sin(5t) + \frac{1}{2} \sin t = -\frac{\sqrt{3}}{2} + \frac{1}{2} \sin(5t) - \frac{1}{2} \sin t$

$$\sin t = -\frac{\sqrt{3}}{2}$$

$$\sin t = -\frac{\sqrt{3}}{2}$$

$$t = \frac{2}{3} (3\pi n - \pi)$$

$$t = \frac{1}{3} (6\pi n - \pi)$$

For the interval $[0, 2\pi)$

$$t = \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$12. \frac{\sin^2(2u)}{\sin^2 u} = 4 - 4\sin^2(u)$$

$$\sin(2x) = 2\sin x \cos x$$

$$\text{LHS } \frac{(2\sin u \cos u)^2}{\sin^2 u} = 4\cos^2(u)$$

$$\cos^2 x = 1 - \sin^2(x) \text{ (using this identity)}$$

$$\Rightarrow 4(1 - \sin^2 u)$$

$$\Rightarrow 4 - 4\sin^2 u$$

$$\therefore \text{LHS} = \text{RHS.}$$

Extra credit

$$1. \left[\cos\left(\frac{x}{2}\right) \right]^2 = 0$$

Apply the rule $x^n = 0 \Rightarrow x = 0$

$$\cos\left(\frac{x}{2}\right) = 0$$

$$\frac{x}{2} = \cos^{-1}(0)$$

$$\frac{x}{2} = \frac{\pi}{2} + 2\pi n, \frac{3\pi}{2} + 2\pi n$$

$$\therefore x = \pi + 4\pi n, x = 3\pi + 4\pi n$$

For the interval $[0, 2\pi]$

$$\boxed{x = \pi},$$

Extra credit

(7)

- 2) Find the exact value of the expression if possible
 $\sin^{-1} [\sin(5\pi/4)]$

Ans

$$\sin 5\pi/4 = -\sin(\pi/4) = -\frac{\sqrt{2}}{2}$$

$$y = \sin^{-1} [-\sqrt{2}/2]$$

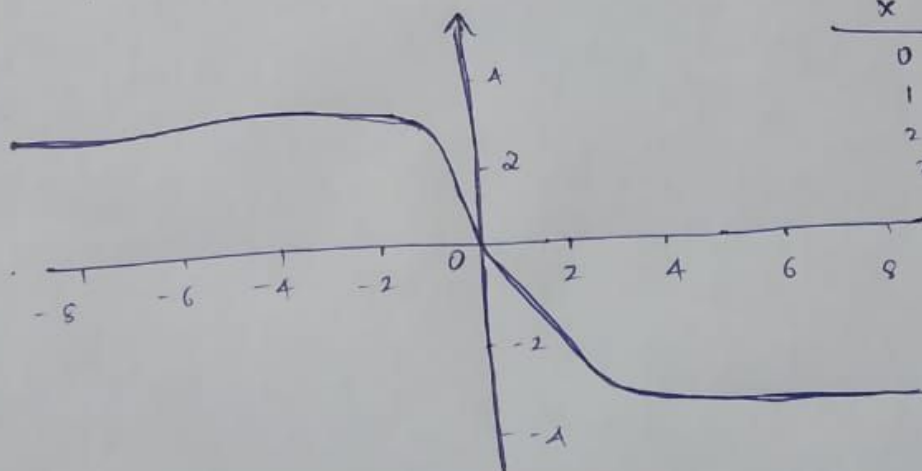
$$-\frac{\sqrt{2}}{2} = \sin(y)$$

In other words, we want to know where the sine function returns $-\sqrt{2}/2$.

Sine is equal to $-\sqrt{2}/2$ for $5\pi/4 + 2n\pi$, $7\pi/4 + 2n\pi$, where n is any integer. But really since the domain of arcsine is $[-\pi/2, \pi/2]$, there is no exact value for the given expression.

- 3) Graph $f(x) = -2 \tan^{-1}(x)$

Ans



x	f(x)
0	0
1	-90
2	-127
3	-143

4) What did you like best about the class? ⑤

Ans

The class is well prepared and informative. Humour in lectures is always appreciated. I liked the instructor's ability to explain concepts and complex problems in a manner that is easy to understand. Also classes were always fun and interesting.

5) What did you like least about the class?

The only thing which was I liked least about the class was the homeworks really were too time consuming. I suggest more smaller assignments.